THENWHATS

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Many years ago, in the days when 'coursework' meant work done during the mathematics course, my colleagues at Peers School and I met to discuss the point at which a piece of work could be described as 'finished' (Watson et al, 1990).[1]

This was not a trivial question, because a 'piece of work' could be a write-up of an exploration which had gone on for one, two, even three weeks or more, and every piece contributed to a folio for eventual assessment. It might not even be a write-up, but a working record, including rough work, which had been collated as the exploration took place – a kind of mathematical log. My then colleague, John Warner, who left teaching rather than conform to more restrictive norms, used to draw a distinction between 'writing-up' and 'writing-down' which captures the difference rather nicely.

Among other features of their work, students were, and sometimes still are, assessed on how they identify and pursue further questions arising from starter tasks. In theory, then, a 'piece of work' could continue for months as, through mathematical enquiry, one thing leads to another. On the other hand, students like to complete work, tidy it up, hand it in, get some feedback and start afresh - and teachers using such approaches like to take home piles of folders and get started on marking the work and using it to assess progress. (More can be read about this way of working in Boaler, 1997 [2]; Ollerton and Watson, 2001[3]; Ollerton, 2002.[4])

My colleagues and I frequently discussed what to say to students who do exactly what has been asked for and then claim to have finished. For example, what if the task had been to find the number of paving slabs needed to surround a certain square pond, given a size of slab? (I have learnt from recent experience that the correct answer is 'more than you think' because an expert landscaper will want to select from a collection of slabs!) Some students draw a diagram, count the slabs, and write a report. Others start writing the report before they do anything towards finding an answer. Huge efforts are put into indicating to such students how they might vary the sizes to get a sense of generality. By and large, school students soon pick up the idea that they are expected to draw several squares, find a formula, and then perhaps (unless the time devoted to that task is over) move on to rectangles of various proportions or otherwise vary the problem. (What about cuboids? What about a circular pond?)

As Dave Hewitt said in MT 140 (1992) [5], for many the task is over when all that has happened is guessing an algebraic expression which matches a table of values and makes the numbers work. Where is the mathematics, he asked?

A pattern has been spotted;
a graph may have been plotted;
a formula is found
or, much more often, passed around.

What we say when students believe they have finished their work, but we want them to probe further, and what Dave asks about the value of 'pattern spotting' tasks have in common is the question 'then what?' This has led me to think more about 'thenwhats'. I want to define a thenwhat as a moment in a lesson when the teacher and/or students have used a strategy which is supposed to be about mathematical thinking, but somehow there is no clear path forward as a result.

In each of these the potential to confuse the *form* of a recommended strategy with its intended *function* in terms of learning and doing mathematics is revealed.

Here are some thenwhats to consider:

- A student is able to describe a general formula which applies to a particular structure and uses it to fill out a table of values for coursework – thenwhat?
- A class have shared five different methods of doing a division calculation in a plenary session – thenwhat?
- A class have been asked to make up questions for which the answer is 5, they have all done so – thenwhat?
- A teacher has asked an open question and written five learners' answers on the board – thenwhat?
- A mental starter intended to last for ten minutes has been interrupted by a student asking 'why don't we have to learn the thirteen times table – it looks just as hard as the seven times?' – thenwhat?

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In other words, the 'thenwhat' is to study the relationships between such formulae and their spatial and graphical representations, their usual and unusual behaviours.

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A student is able to describe a general formula which applies to a particular structure and uses it to fill out a table of values using for coursework – thenwhat?

The student believes that the table of values is an essential part of the work, but for him the table is of no value as he already sees a generalisation. What is intended to be a tool for generalisation has become some kind of ritual in its own right. Some recent research by psychologists into induction methods shows that systematic generation of values does not necessarily contribute to correct induction – insight and tenacity are more influential (Haverty, 2000) [6]. As a teacher, I might celebrate the student's insight by suggesting finding some unusual values, to relate the formula to the original data, to explore what might happen with negative numbers, to invent structures which generate related formulae. In other words, the 'thenwhat' is to study the relationships between such formulae and their spatial and graphical representations, their usual and unusual behaviours. What matters is not the formula but how it was constructed from the situation.

A class have shared five different methods of doing a division calculation in a plenary session – thenwhat?

You might have said 'then the bell goes'! Sharing can extend students' knowledge of what is possible, but they would probably have to do some further work to grasp what is being offered. The focus of such a lesson has shifted from answers to methods but this new focus can dissipate unless the methods are then used to make comparisons. Which methods are appropriate for which kinds of numbers? Which are most efficient? Which are easiest/hardest and why? Which are easiest to record? Does recording help with accuracy? Unfortunately, sharing at the end of the lesson is likely only to fulfil the social function of making people feel successful and involved, or the assessment function of letting the teacher know what has gone on. The form of sharing has been used, but its function in mathematical learning may be absent.

A class have been asked to make up several questions for which the answer is 5, they have all done so – thenwhat?

The beliefs behind this strategy are that making up your own questions helps you answer other people's, and that working backwards helps you understand concepts better than performing algorithms would do. But it is still possible for a student to make this into a trivial task by choosing obvious and easy options. It is also not clear from research that generating questions and giving them to peers to answer *does* help answer other people's questions, although it often gives teachers some assessment information and motivates students to work. Once again, it is possible to confuse the form of question-posing with its function, which is engagement with mathematical concepts. In one class a student wrote

$5+0=$
$0+5=$
$1\times5=$
$5\times1=$

No one could leave that collection just dangling in the air! For example, one could discuss using '5' as a placeholder for a generality. Another way forward is to make the questions a focus for comparison and discussion, so that students have to choose their favourite/hardest/most unexpected question from those produced and have to say why.

Another is to put some constraints on question posing so that students have to explore concepts in order to produce what is required. For example, 'the answer is 5, there is at least one negative number involved, you cannot use add, you must use every digit at least once, there must be two fractions involved . . .' etc. This, of course, turns a quick activity into something which might need a lot of thought and calculator work.

A teacher has asked an open question and written five answers on the board – thenwhat?

I am becoming less and less entranced by the distinction between closed and open questions. Of much more interest is whether a sequence of questions opens-up or closes-down possibilities for a learner, and whether such opening or closing is helping them learn some mathematics. For example, the question 'give me a question whose answer is 5' is very open and consequently can be rather uninteresting. The gradual closing-down suggested above makes it more interesting, more challenging, more mathematical. The closing-down of some possibilities opens-up others. The more closed it gets, the more new possibilities are offered – although there may come a point after which more closure makes things too hard and meets resistance. The purpose of open questions is to encourage thinking and participation, but not all open-ended questions achieve this, while some closed questions can generate a great deal of thought. Participation in unstructured open answers can be merely social. It is the space to produce various answers within constraints which challenges students to be mathematical.

A mental starter about 'difficult' times tables, intended to last for ten minutes, has been interrupted by a student asking 'why don't we have to learn the thirteen times table – it looks just as hard as the seven times?' – thenwhat?

References

1 A. Watson et al: *Finishing mathematical work*. ***MT130*** 45-47, 1990
2 Jo Boaler: ***Experiencing school mathematics***. Buckingham: Open University Press, 1997
3 M. Ollerton and A.Watson: ***Inclusive mathematics 11 – 18***. London: Continuum, 2001
4 M. Ollerton: ***Learning and teaching mathematics without a textbook***. Derby: ATM, 2002
5 D. Hewitt: *Train spotters' paradise*. ***MT140***, 6-8, 1992
6 L. A. Haverty et al: *Solving inductive reasoning problems in mathematics: not-so-trivial pursuit*, ***Cognitive science*** vol.24 (2) 249-298, 2000

This mental starter has been successful in starting up someone's mental powers but if the teacher chooses to follow this line of questioning the starter is going to last well beyond ten minutes, and the planned lesson may have to be abandoned. The function of mental starters is to stimulate thinking and get some fluency into the way students access their knowledge. The form – ten minutes of activity, particularly if it is unrelated to the rest of the lesson – can get in the way of this function. I would probably have asked the student to elaborate on his question, and then asked the class what they would like to do with it. Some questions are more interesting than my lesson plans.

Planning for thenwhats

My purpose in writing this has been to show how some strategies designed to stimulate mathematical learning and thinking can sometimes go nowhere if we do not think about where to go next, or how to use what happens as a result of the strategy. In all the examples I have given, the 'thenwhat' arises because the classroom has become entangled in 'doing' things in a particular way, rather than in using what is done to provide material for further consideration. The existence of a thenwhat marks an *opportunity to change gear*. In each of the scenarios described above, there is the potential to change from paying attention to the initial 'doing' to thinking about the results of that 'doing' as a class of objects. For example, there could be a shift from getting answers to comparing, contrasting, sorting and generalising methods; a shift from getting one formula to exploring the class of similar formulae; a shift from doing multiplications to looking at multiplicative structures of numbers; a shift from social participation to mathematical participation.

When trying to predict the kinds of responses students will give to tasks we could also be asking 'thenwhat?' and identifying how the lesson could change gear and become more general, more abstract, more mathematical.

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