CHAPTER 4
SILENT MULTIPLICATION

Overview

The goal of this activity is for students to explore patterns in factors and products in order to help them develop understanding of the mathematics underlying multidigit multiplication. Students develop strategies to solve increasingly complex problems by using what they already know about simpler related problems. For example, students examine the effect on the product when one factor is doubled, when both factors are doubled, when one factor is halved, when one factor is halved while the other is doubled, and so on. Students also explore the effects on the product when multiplying by 10, and multiples of 10. As students gain facility and confidence with this activity, they can reverse the process by taking a complex problem and solving it by making it into simpler related problems. Skill with computation, number sense, and problem solving are employed and strengthened as students experience this activity. The word *silent* in the title indicates the teaching method used for the lesson, as described in the following sections.

Materials

▲ none

Time

▲ two class periods followed by shorter explorations several times per week spread over many weeks

Teaching Directions

1. Explain to students the rules for a silent lesson:

▲ A star drawn on the board or overhead indicates the beginning of the activity and silence by everyone, including the teacher.

▲ When a problem is written on the board, students should indicate when they know the answer by putting their thumbs up.

▲ When an answer is written, students should indicate agreement with thumbs up, disagreement with thumbs down, or indecision or confusion with thumbs sideways.

2. Draw a star on the board or overhead transparency to indicate that it's time for silence. For an introductory experience, write a multiplication problem on the board or transparency that all students can solve; for example *1 × 2.*

3. Wait for students to show thumbs up.

4. Hand the chalk to a student and indicate that he or she should write the product on the board. Wait for the other students to indicate agreement, disagreement, or indecision or confusion by putting their thumbs up, down, or sideways.

5. Write a second related problem (for example, *2 × 2*) under the first problem. Again wait for the students to indicate when they know the answer with their thumbs. After most students indicate they know the answer, hand the chalk to a volunteer to write the answer on the chalkboard. Have the other students indicate with their thumbs agreement with the answer, disagreement, or indecision or confusion. Erase the star, indicating talking is permitted.

6. Lead a discussion about how the two problems are related and how students can use what they know from the first problem to help them solve the second problem.

7. While steps 1 through 6 model for students the basic structure of this activity, emphasize to the students the need to be silent and think about how to apply what they already know to solve each new problem. Draw a star and continue with the silent lesson for a series of four or more related problems.

8. Erase the star and lead a class discussion about how students used what they knew about one problem to solve another. Pose such questions as the following:

How are these two problems related?

What is the same about these problems?

What is different?

What happened to the factors?

What happened to the products?

9. On Day 2, teach the silent lesson again, exploring a new idea such as multiplying by 10. Leave time at the end of class for students to respond to the following prompt: "Something I learned playing *Silent Multiplication* is . . . "

10. Continue on other days with other sequences of problems. See pages 41, 43, 45, 47, and 48 for suggested sequences.

Teaching Notes

This activity works well when used several times a week over an extended period of time. It gives students opportunities to explore and gain understanding about factoring numbers, using the commutative and associative properties of multiplication, using the distributive property, and multiplying by 10, powers of 10, and multiples of 10.

A lesson can focus on just one idea at a time, multiplying by 10, for example. The nature of our number system is such that to multiply a number by 10, you simply need to add a zero, resulting in a zero in the ones place of the product: $10 \times 4 = 40$, $112 \times 10 = 1,120$, and $23,490 \times 10 = 234,900$. Another focus for a lesson might be multiplying by multiples of 10. For example, to solve 20×5, the problem can be thought of as $2 \times 5 \times 10$. When thinking of a problem in this way, the student is developing understanding about factoring numbers and the associative and commutative properties of multiplication. At another time the lesson might focus on what happens to the product when factors are doubled or halved: doubling one factor while the other remains the same results in the product doubling ($4 \times 8 = 32$, $8 \times 8 = 64$); doubling both factors results in the product quadrupling ($2 \times 2 = 4$, $4 \times 4 = 16$); halving one factor while the other remains the same results in the product halving ($8 \times 6 = 48$, $4 \times 6 = 24$); halving both results in a product being divided by four, or one-quarter of the original product ($8 \times 6 = 48$, $4 \times 3 = 12$); halving one factor and doubling the other causes the product to remain unchanged ($18 \times 10 = 180$, while 9×20 also equals 180).

Understandings and insights from these lessons support students as they grapple with the task of efficiently and accurately finding products of multidigit multiplication problems. With time and experience, students learn to solve difficult problems by solving simpler related problems. For example, to solve 240×12, a student might think of 240 as $12 \times 2 \times 10$. With this in mind, the student can then solve 240×12 mentally by multiplying $12 \times 2 \times 10 \times 12$ to get the product 2,880. While the student may not recognize it, the way in which these numbers are actually calculated may involve factoring and applying the commutative and associative properties. Or a student might solve 240×12 by thinking of it as $(240 \times 10) + (240 \times 2)$, and then adding $2,400 + 480$ to get 2,880, an example of the student using the distributive property upon which the traditional algorithm used in the United States is based.

Of critical importance in this activity is the discussion following a series of problems about how the problems are related. By understanding how two problems are related and by using what they already know, students can move forward to solve new, unknown, more complex problems.

An unusual characteristic of this activity is that students and teacher participate silently in some parts. Students indicate when they have solved a problem mentally and their agreement or disagreement with an answer by putting their thumbs up to indicate agreement, down to indicate disagreement, or sideways to indicate indecision or confusion. The silent aspect provides a nice change of pace to typical classroom instruction and prevents blurting out, giving students additional think time. Children also enjoy the opportunity to come to the board to write their responses, and the silent experience provides a basis for valuable mathematical discussion.

When this activity was introduced to the students in the vignette that follows, their experience had been primarily limited to single-digit multiplication, although a few students knew about what happens when any number is multiplied by 10. Thinking about halving and doubling was difficult for these students. As the students became more comfortable with the idea of doubling, they began to incorporate the idea of halving, although the notion of halving was more difficult.

This activity can easily be adapted to the skill level of the students. In some classes, students have asked questions that provided a nice starting point for the activity. When students don't offer a place to begin, a question can be posed for them to explore as they move through the activity. In the following vignette the question posed was "What happens to the product when one factor is doubled?" The vignette illustrates a particular direction this class took. Your students will raise different questions and have different insights. The important ideas on which to remain focused throughout the discussion are the patterns and relationships among the problems and how this information can help students solve more difficult problems.

Additional lessons that support some ideas presented in *Silent Multiplication* are *Related Rectangles* and *Target 300*. *Related Rectangles* links number patterns explored in *Silent Multiplication* with the geometric interpretation of multiplication as rectangular arrays. *Target 300* provides students with additional practice with multiplication by 10 and multiples of 10.

The Lesson

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DAY 1

I explained to the class, "I'd like to do an activity with you in which you'll look for patterns. I'll start by writing a simple problem on the board that I think you all will know. When you know the answer, show me by putting your thumb up. I will hand someone the chalk and that person may come to the board to write the answer. If I hand you the chalk and you don't want to come to the board, just shake your head no and I will give the chalk to someone else. If you agree with the answer that gets written on the board, put your thumb up. If you disagree, put your thumb down. If you are not certain, put your thumb sideways. Here's the unusual thing about this game: it is played in silence! No one talks, not even me! I will put a star on the board to indicate we must all be silent. I will erase the star when we can talk again." There were no questions, so I decided to start the activity. I made the first round easy so everyone would be successful and become clear about the rules. I began by drawing a star

on the board to indicate to all that it was time to be silent. I wrote *1 × 2 =* on the board. Immediately all thumbs shot up. I handed the chalk to Rachel, who shook her head no. I quickly handed the chalk to Alex, who came to the board and wrote *2.* All students indicated their agreement with Alex's answer by putting their thumbs up. Next I wrote *2 × 2.* Again thumbs were up immediately. I handed the chalk to Sam, who wrote *4.* I erased the star, indicating we could talk.

"Who would like to share their thinking about their answer to the first problem?" I began. I wanted the students to see immediately that the problems used for this activity were related. I wanted them to look for the relationship and use that relationship to help them solve move complex problems.

"It was easy. The answer is two. The problem means one group of two, which is two," Sarita responded. Several students nodded, indicating their agreement.

"What about the second problem? What is alike about the two problems and what is different?" I probed.

"Each of the problems has one factor that is two," Nicole said.

"And for the answer you just added the answer from the first problem to itself to get the second answer," Steve explained.

"That's doubling," Allie noticed.

"Oh yeah!" Steve said.

"One factor is the same and the other changes," Shelly added.

"How did it change?" I asked.

"Well, both problems have a two, like Nicole said, but with the other factor, it's like you added it to itself to get the new factor," Shelly explained.

"That's doubling," Allie repeated again.

"So, to get to the second problem from the first one, one factor doubled, one factor stayed the same, and the product doubled," Cori summarized.

"If you agree with Cori, put your thumb up, if you disagree, put your thumb down, if you aren't sure, put your thumb sideways," I said.

Most children indicated their agreement while a few indicated that they weren't sure. I decided to continue the activity and keep a close eye on the students who indicated their uncertainty, hoping that with further exploration, the mathematics would reveal itself.

I put the star back on the board, indicating that silence was necessary. I continued this round with the following set of problems:

$4 \times 2 =$

$8 \times 2 =$

$16 \times 2 =$

$32 \times 2 =$

This series of problems was easily accessible for all students. I erased the star, indicating it was time to talk about what we had just done. "How did the first problem help you solve the second?" I asked.

"Well one times one equals two and so you just add the answer twice to get the answer for the next problem," Jackie explained. "It's that way the whole way. You just add the answer twice to get the new answer." While Jackie showed that she was seeing a pattern in the answers, or products, I was concerned because she indicated no understanding of what was causing this pattern. She seemed to be focusing on the products without looking at the factors or considering why the products were doubling.

"Why do you think the products are doubling?" I asked.

"It's just the pattern I see," she replied.

"I noticed something. One of the factors is always staying the same. The other one is always timesing by two, like Cori said about the first two problems," Juan said.

"I think Juan is right. I noticed that the first factor doubles. It's like if one factor

doubles then the product doubles," Rachel shared. The others indicated their agreement with what Rachel and Juan had shared.

"Let's try some more and see if it always works," David suggested.

I used the following sequence for our next round. In the first series, the first factor always doubled. This time I switched which factor doubled so that sometimes it was the first factor and sometimes it was the second.

$2 \times 2 =$

$2 \times 4 =$

$4 \times 4 =$

$4 \times 8 =$

$8 \times 8 =$

$16 \times 8 =$

$16 \times 16 =$

This doubling pattern involved slightly larger numbers than the first series but still was accessible to the students. Taking small steps in the beginning helps ensure success while creating confidence and a greater willingness in students to take risks when the learning gets more challenging.

After the last problem I again erased the star and we had a similar discussion to the first. "What did you notice?" I began.

"It's like you add a factor twice when you go from one problem to the next. Then you add the answer from the first problem twice to get the answer to the next one," David shared.

"It seems like sometimes you used the product from before to get one of the factors in the new problem," Anamaria said. "Like two times two equals four, the product four became one of the factors in the next problem, two times four. But not with two times four equals eight and then four times four equals sixteen. But you could have used the eight and done two times eight equals sixteen instead of four times four equals sixteen!"

"It changes; it's getting bigger," Amy added.

"It's doubling," repeated Allie for the third time, disgusted that her observation was not being understood by her classmates.

"I get it! It's doubling! My hypothesis is that if one factor doubles and one stays the same then the product doubles!" James said, lighting up. Allie rolled her eyes. This reminded me that even though I may tell students something, it does not mean that they hear it in a way that means they understand it or can apply it. I am reminded that each child must make meaning and sense of learning for her- or himself. Understanding is not something that can be gained by simply being told; rather, it is gained through thought and interacting with ideas and experiences.

"Why do you think this is happening?" I asked the students. The students sat quietly, thinking about why.

"How could I represent one times one with a rectangular array?" I continued.

"It would be a rectangle, a square really, that is one going vertically and one going horizontally," explained Sam. I drew what Sam described. I continued the discussion in this fashion, hoping that by giving students the opportunity to look at the geometric representation of what was happening, more would gain understanding of why doubling one factor doubles the product.

$2 \times 4 = 8$

$2 \times 8 = 16$
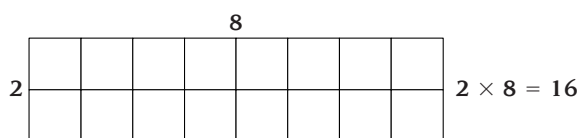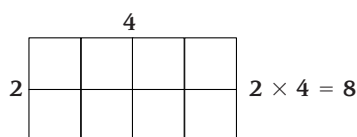
"I see what is happening. One side of the rectangle is staying the same and the other side of the rectangle doubles! That's cool!" David said.

"What if you use odd numbers? You have mostly used even numbers. If you used odd numbers and doubled one factor, will the answer still double?" Ben wondered.

"I don't really think odd and even matters. I think it's the doubling part that matters. If you do something in one part of the problem, it changes things in another part. So it's the doubling, not if it's odd or even," Daniel replied after a few moments of thought.

Nicole raised her hand with excitement. "I think I discovered something! I have a hypothesis! Whenever a factor is doubled, odd or even, the product is always even. Like three times three equals nine, but double one of the threes to make six and six times three equals eighteen!"

"Nicole, Daniel, and Ben have some interesting ideas. Let's try a few more rounds of *Silent Multiplication* and see if we can get some information to help them prove or disprove their ideas," I replied.

Cori summarized the discussion and made a request. "Well, Nicole and Ben are sort of the same. Ben is wondering about odd numbers and Nicole is saying that odd numbers when doubled are even. I don't know if she is right. Adding two odd numbers makes an even number, adding three odd numbers makes an odd number. And doubling an odd number is like adding the same odd number twice, which is like multi-

plying by two. Addition and multiplication are related because you can do repeated addition if you don't know the multiplication answer. But I am not sure. Can we use odd numbers this time?" The rest of the students indicated their interest and agreement.

After putting the star on the board to indicate time for silence, I gave the following sequence of problems:

$1 \times 3 =$

$2 \times 3 =$

$4 \times 3 =$

$8 \times 3 =$

$8 \times 6 =$

$6 \times 8 =$

$6 \times 16 =$

$16 \times 12 =$

I erased the star and hands jumped into the air. To give as many students as possible an opportunity to share their ideas, I asked the students to each share their thinking with a neighbor. After each partner had had thirty seconds to share her or his thinking, I asked for the class' attention once again.

"I think my hypothesis was right. If you double any number, odd or even, it will always end up even," Nicole shared with excitement.

"Have you noticed that any number times two is always an even number?" Tom asked. "And doubling is like multiplying by two. That's why Nicole's idea works."

"Hey, I was just thinking. What about multiplying a fraction by two. Then it doesn't always work. Like one-half times two equals one, an odd number, or one-and-a-half times two equals three, another odd number!" Shelly pointed out.

"OK, I see your point. Then any *whole* number times two equals an even number!" Nicole clarified. "And I think two odd numbers multiplied together make an odd

number, like three times three equals nine and five times seven equals thirty-five. But doubling an odd number makes an even number, like Cori said, because it's like adding an odd plus an odd, which is an even, it's just you are adding the same odd number twice." Cori nodded her agreement along with most of the other students.

"Let's look at the problems and see how they are related and how knowing something about one can help us solve the next one," I said, turning the conversation in a different direction.

"Let's look at one times three equals three and two times three equals six. What do you notice about these problems?" I asked.

"Well, one factor is the same in both," Tom explained. "That would be the three. And then one factor in the second problem doubled from the first one. One doubled to two. And the products are different, too. Three doubled into six. If you think about rectangular arrays, it is like you had one row of three for the first problem and then you added a second row of three for the second. You doubled the amount of rows and the total number of squares in the array doubled, too," Tom explained. To help the class understand Tom's explanation and to verify that I understood his thinking, I drew the following on the board:

Problem 1
1 row of 3
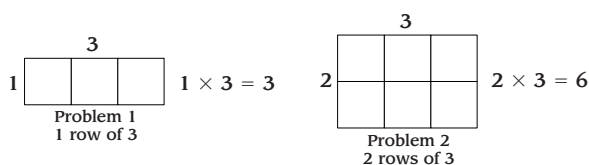
Problem 2
2 rows of 3

"Is this what you mean?" I asked. Tom indicated that my drawing represented his thinking. (For more on using rectangles with multiplication, see Chapter 5, "Related Rectangles.") "What about the second problem, two times three equals six, and the third problem, four times three equals twelve?" I continued. I called on Jackie.

"I already knew that four times three equals twelve. You can prove it by counting by threes four times or counting by fours three times. Or you can add four plus four plus four or three plus three plus three plus three. Or you could draw a rectangle that was four going down and three going across. Or you could notice that two times three equals six, and four is two times two, so since two was multiplied by two to get four then you could multiply six times two to make twelve!" Jackie's initial understanding concerned me, but here she recognized an important mathematical idea, that is, what you do to one side of the equation, you must do to the other side of the equation.

"What about four times three and eight times three? What is going on there?" I continued.

"It's the same thing; one of the factors doubled so all you had to do was double the last product. Four doubled into eight, so twelve has to double into twenty-four," Cindy answered.

"What about eight times three and eight times six?" I asked.

"It's quite simple really. It's the same thing again. One factor doubled, the three became a six, so the twenty-four had to go twenty-four plus twenty-four to make forty-eight," Shelly replied.

"The next one didn't do anything except switch the order. Instead of eight times six it got switched to six times eight," Daniel said. I had included this pair of problems to give students experience with the commutative property of multiplication. In previous discussions, we had noted that a picture of eight groups of six indeed looks different than six groups of eight, but the final product is the same. Students were familiar with this piece of information because it was useful in helping them learn their basic facts.

"If you look at the next several problems, it's the same thing as doubling one factor, one staying the same, and the prod-

uct doubling," Steve said. I asked the students to indicate their agreement or disagreement with Steve with their thumbs. They indicated their agreement.

I recorded the students' observations as follows:

$1 \times 3 = 3$

$2 \times 3 = 6$ *double factor × same factor = double product*

$4 \times 3 = 12$ *double factor × same factor = double product*

$8 \times 3 = 24$ *double factor × same factor = double product*

$8 \times 6 = 48$ *same factor × double factor = double product*

$6 \times 8 = 48$ *commutative (order) property = same product*

$6 \times 16 = 96$ *same factor × double factor = double product*

$16 \times 12 = 192$ *same factor × double factor = double product*

"Can we do one more?" David asked. Again, I drew a star on the board to indicate silence was needed. The students immediately were quiet, with their attention on me and the board. I decided for the final series of the day to stick with the doubling pattern we had been exploring, but this time I decided to use numbers that were somewhat more challenging:

4×3

4×6

4×12

4×24

8×24

16×24

32×24

I wrote $4 \times 3 =.$ Immediately thumbs went up. I handed the chalk to Alex, who came to the board and quickly wrote *12.* His classmates indicated their agreement by putting their thumbs up.

Next I wrote 4×6 and again thumbs went up quickly. Cindy came to the board and wrote *25.* Her classmates gave her a quick thumbs down. She took another look at her work and immediately revised her answer to *24,* which received a thumbs up from the students.

The game continued with 4×12. The students were not as quick to put their thumbs up on this one, so I gave them a bit more time. When most had their thumbs up, I handed the chalk to Allie, who wrote *48,* to which the students gave a thumbs up.

The next problem proved more challenging still for the students. The child who came to the board got stuck. The problem was 4×24. I handed the chalk to Shelly. She wrote the product of 4×24 as *916.* Most of the other students gave her a thumbs down. She looked puzzled. I drew an arrow from the 12 to the 24 and wrote *same or different?,* to which she wrote *different.* I wrote *How?* Shelly responded *Double* then to the side, she wrote *48 + 48?,* indicating that she knew 48 from the previous problem needed to be doubled. I nodded yes. She then worked the problem on the board, getting the answer of 96. The others indicated their agreement with a thumbs up. Shelly's initial response of 916 was a red flag to me that Shelly was not using number sense and an indicator that class discussions needed to focus more on reasonable answers.

I wrote 8×24 next. Juan came to the board and wrote *192.* The students agreed. The next problem, 16×24, got a wide-eyed response from the class. After a moment to analyze the situation, however, thumbs were soon up, indicating that students thought they had an answer. Ben came to the board and wrote *384.* The students agreed.

The next problem was 32×24. Grinning faces and thumbs up indicated the students

were gaining confidence in themselves. Anamaria wrote *768* as the product. A few students indicated they weren't sure by turning their thumbs sideways, but most agreed. The final problem of the series was 24×32. All thumbs were up immediately, as the students recognized I had simply used the commutative property and switched the order of the factors, causing no change in the product. A discussion of how you can use what you know from one problem to help you solve the next followed.

"What did you think of this activity?" I asked after the discussion.

"It was really cool!" James said.

"What made it cool?" I asked.

"Everyone had to think and then someone had to write the answer on the board and then we got to tell if we agreed or not," James explained.

"What did you learn from doing this activity?" I asked.

"We discovered that if you double a factor, the product doubles, if the other factor stays the same," Cori said. "It's like if you do something to one side of the problem, you have to do the same thing to the other side, too. I wonder what happens if both factors double?"

"I don't think the product would double then because that is what it did when only one factor doubled. Now two factors are doubling," James said.

"What happens if you multiply one of the factors by ten?" wondered Tom.

"What if you doubled one factor and multiplied the other one by ten? Then what would happen?" Nicole asked.

I was delighted that the students were curious about these questions. I wrote their questions on the board and told them these were some ideas we would explore another day.

"I was amazed I could figure out such hard problems!" Jeni added with surprise.

"In a way the problems got harder as we went along, but in a way not, because we could use the answer from the problem before and just double it," Sandra said.

"It's just looking for a pattern from one problem to the next and using it," Alan said.

DAY 2: EXPLORING MULTIPLICATION BY 10 AND MULTIPLES OF 10

The enthusiasm of the students from the previous day continued into the second day with the students asking if we could do *Silent Multiplication.* My goal was to give the students the opportunity to explore multiplication by 10 and multiples of 10. Because of the nature of our number system, when a factor is multiplied by 10, the result is that factor with a zero added to the end, or in the ones place. For example, $4 \times 10 = 40$, $432 \times 10 = 4,320$, and $6,120 \times 10 = 61,200$. When one factor is multiplied by a multiple of 10, students can factor and make use of the associative and commutative properties to help them more easily and efficiently solve the problem. For example, 20×56 can be thought of as $2 \times 56 \times 10$. The 20 has been factored into 2×10. The commutative property of multiplication, which has to do with changing the order of the numbers, was used when the 2 was placed at the beginning of the string of factors and the 10 at the end. The associative property of multiplication has to do with how numbers are paired. The student could think $(2 \times 56) \times 10$ or $2 \times (56 \times 10)$. When students understand and apply these ideas, they can become more efficient and accurate in their computation of multiplication. These ideas are also foundations for their later study of mathematics.

I began the lesson with a quick review of what we had done the day before. This benefits both the students who were there as well as those who were absent. I drew

the star on the board and used the following series of problems:

1 × 10 =

2 × 10 =

4 × 10 =

6 × 10 =

12 × 10 =

14 × 10 =

16 × 10 =

23 × 10 =

32 × 10 =

Students had no problem with the first four problems. However, 12 × 10 proved more difficult for some, including the student who came to the board. Tom thought the product was 110. When he received the thumbs down from his classmates, he paused a moment, then began to use his fingers to keep track as he silently counted by ten twelve times. He revised his answer to 120. The students were also somewhat hesitant with 14 × 10. Cori came to the board and wrote 140. This seemed to reaffirm their somewhat tentative thinking and they seemed more confident for the last three problems, even excited, as they saw the "big" problems they were able to solve mentally.

As shown previously, a double-digit factor multiplied by 10 gave some students difficulty. Thinking of 12 as 10 + 2 can help students with this difficulty. When students have this understanding, they can think of the problem 12 × 10 as (10 × 10) + (10 × 2), a use of the distributive property. This is foundational for later topics in mathematics and the traditional algorithm. This insight also supports the continued development of students' number sense.

"Yesterday you noticed that you could use what happened in one problem to help you solve the next. What was the pattern we were looking at yesterday?" I asked

after erasing the star and beginning the discussion.

"Yesterday we found out if you double one factor and one factor stays the same, the answer doubles," Anamaria responded. The other students put their thumbs up to indicate their agreement.

"Was that what was happening today?" I asked.

"Sort of. In the first two problems, the ten was the same, but the other factor doubled, so did the product," Tom noticed.

"That's the same thing that happened with two times ten and four times ten," Allie said.

"But I notice something else. The product is the same as one of the factors, only a zero was added at the end," Anamaria shared.

"I noticed that, too. Then I figured out I could just count by tens however many times the other factor was. And then I thought about counting by ten and noticed that if I count by tens, then no matter how many times I count I always end up with a number that has zero in the ones place," Cori said.

"What if you count by tens and begin with thirteen? Then it doesn't work," replied Nicole. "If you start with thirteen then it goes twenty-three, thirty-three, and the ones place always has a three in it."

"Yeah, but if you count by tens mostly you start with ten and then it works," Cori responded.

"It looks like you can figure out some problems two ways. A lot of the problems have a factor that doubled from the one before, so you could just double the product from the one before. Or you could just take the factor that isn't ten and add a zero in the ones place," Shelly summed up.

"If it doesn't double, you had to know about multiplying by ten and adding the zero. If you knew that, it was easy! If not, counting by ten thirty-two times like in the last problem would take a really long time!" Rachel said.

I decided to try one more series with the students since they were picking up new ideas quickly and were engaged in and enjoying the activity. I decided to include the idea of halving and the idea of multiplying by multiples of 10. I used the following problems:

$10 \times 10 =$

$10 \times 5 =$

$20 \times 5 =$

$40 \times 5 =$

$40 \times 10 =$

$80 \times 10 =$

$80 \times 5 =$

The activity proceeded as it had in the past. After a few more series in which we explored doubling, halving, and multiplying by 10 and multiples of 10, I asked the students to quickly write about what they had learned from this activity.

SUGGESTIONS FOR ADDITIONAL EXPLORATIONS AND DISCUSSIONS

Goal: doubling one factor

$3 \times 4 =$

$4 \times 6 =$

$4 \times 12 =$

$4 \times 24 =$

$8 \times 24 =$

$24 \times 16 =$

$32 \times 24 =$

Goal: multiplying by 10

$4 \times 1 =$

$4 \times 10 =$

$40 \times 10 =$

$41 \times 10 =$

$45 \times 10 =$

$451 \times 10 =$

Goal: halving and doubling

$2 \times 6 =$

$2 \times 12 =$

$4 \times 12 =$

$4 \times 6 =$

$8 \times 6 =$

$8 \times 3 =$

Goal: doubling, halving, and multiplying by 10 and multiples of 10 adding one more group

$4 \times 6 =$	$6 \times 3 =$
$2 \times 6 =$	$3 \times 3 =$
$20 \times 6 =$	$30 \times 3 =$
$40 \times 6 =$	$15 \times 3 =$
$40 \times 60 =$	$150 \times 3 =$
$80 \times 60 =$	$150 \times 30 =$
$800 \times 60 =$	$151 \times 30 =$
$801 \times 60 =$	$152 \times 30 =$
$802 \times 60 =$	

Goal: doubling both factors—quadruple product multiplying by 10 and multiples of 10

$3 \times 5 =$

$6 \times 10 =$

$12 \times 20 =$

$24 \times 40 =$

Goal: multiplying by 10 and multiples of 10

$1 \times 12 =$

$10 \times 12 =$

$10 \times 24 =$

$20 \times 24 =$

$40 \times 24 =$

Goal: halving and doubling multiplying by 10 and multiples of 10

$4 \times 20 =$

$2 \times 20 =$

$20 \times 20 =$

$200 \times 20 =$

$400 \times 20 =$

$400 \times 40 =$

$200 \times 40 =$

Note: Halving is difficult for many students. Keep the numbers small at first to increase the likelihood students will more easily see this relationship.

After a great deal of experience, give students a more difficult problem and ask them to change it into easier related problems. One day I put the following problem on the board: *16 × 8*. "How could you solve this problem?" I asked.

"You could think of it as an easier problem like sixteen times ten equals one hundred sixty and then subtract thirty-two because there are two groups of sixteen too many and get one hundred twenty-eight," Shelly shared.

"Another way is to make it into two problems. You could do eight times eight times two. You can think of it that way because the eight times two is sixteen, one of the original factors, and then the other eight is the second factor. It's *Silent Multiplication* going backwards!" explained Anamaria. Anamaria used the ideas of factoring numbers to make a simpler problem as well as applying the associative and commutative properties of multiplication.

"You could also do it by changing the sixteen into ten plus six and then multiplying both the ten and the six by eight. Ten times eight is eighty and six times eight is forty-eight. Eighty plus forty-eight is one hundred twenty-eight," David said. David made use of the distributive property.

This is the kind of flexible thinking and making sense of multiplication I had hoped my students would achieve. We continued to explore other similar problems, such as 14×48.

EXTENSION

In another class that had experience exploring patterns in factors and products through

Silent Multiplication, I changed the whole class lesson to a small group activity. I said to the class, "We are going to change *Silent Multiplication* slightly. You are going to play with just your table group. One person will write a problem on a sheet of paper and pass the paper on to the next person. The next person will write the answer and write a new related problem and pass it to the third person. The third person will check the answer the second person gave, solve the second person's problem, and write another related problem and so on. If you think an answer is incorrect, you must prove it to your group. Remember you must do this silently," I explained.

I decided to model the activity quickly. I asked the class to gather around a table. I drew a star on the board, indicating everyone needed to be silent, and sat down. I began by writing *4 × 4 =* and then passing the paper to Alicia. She answered my problem of 4×4 by writing *16* and then wrote *8 × 4.* She passed the paper to Jamie, who checked Alicia's answer of 16, answered Alicia's problem of 8×4 with *28* and wrote the new problem *8 × 8.* Jamie passed the paper to Gina. Gina checked Jamie's answer of 28 for 8×4 and went on to solve the next problem. Alicia noticed that Jamie had written the wrong answer and Gina had not noticed, so Alicia interrupted Gina by tapping her pencil to get Gina's attention. Alicia pointed to $8 \times 4 = 28$ and shook her head no. Then Alicia wrote on the paper *8 × 2 = 16* and *16 × 2 = 32.* Gina and Jamie did not seem convinced. Next Alicia wrote *8, 16, 24, 32.* Gina, who could be a bit stubborn, still shook her head in disagreement with Alicia, so Alicia drew four circles with eight tally marks in each and finally convinced both Jamie and Gina of her thinking. I decided to stop the activity here, as the other students were getting excited and were eager to return to their seats to get started. I erased the star and said, "I

hope you noticed how the girls were able to disagree and justify their thinking. Remember to check each other's work. Are there any questions?" I said.

"Can we get started now?" Conner asked eagerly.

I nodded yes and the students quickly returned to their seats and got to work.

Observing the Students

Watching the students work together was very revealing. By watching closely as the students worked, I was able to gain insights not only about who did and did not understand the mathematical ideas underlying this activity but also about the depth of understanding of many of the students as well as where some of their difficulties were.

Because Susanna and Neal had both had some difficulties with *Silent Multiplication* lessons, I decided to pay close attention to them initially. I checked in with Neal and his group. The group started with $5 \times 30 = 150$. The second person wrote $10 \times 60 = .$ Brooke responded with *600* and wrote *40 × 60.* Neal quickly wrote *2,400* as the product and added the new problem of $80 \times 120 = .$ Charlie responded *9,200.* Neal disagreed with the answer Charlie gave and wrote *12 × 8 = ?? Check the chart.* The chart Neal referred to was the class multiplication chart made of rectangles. Charlie checked the chart and corrected his answer to read *9,600.* (See Figure 4–1.) I was satisfied that Neal understood what he was doing and pleased to see the students use the chart as a way to justify their thinking.

I moved on to Susanna's group. They were having some difficulty in part because of Susanna's confusion. Kirk wrote the first problem, *2 × 3 =,* to which Susanna responded *6.* When Susanna had to write a new related problem, she became frustrated. She wrote *3 × 5,* to which Kirk wrote a long note showing why he felt that 3×5

▲▲▲▲▲▲Figure 4–1 *Neal disagreed with an answer and suggested Charlie check the multiplication chart.*

was not related to 2×3. Susanna erased her work and stared at the paper a moment. Jeni took the paper and wrote, *You know how to double,* to which Susanna responded by erasing Jeni's comment and writing her own, *I don't know!* Miguel, who had been watching but not getting involved, used his fingers. He showed Susanna one finger on his left hand then showed her two fingers on his right hand and wrote on the paper 1 doubled is 2. Susanna also erased this comment, but she wrote the following related problem, *4 × 6 =,* and passed the paper. I wandered on with the intent of returning shortly to check on progress. I appreciated the way the students made an honest effort to help Susanna and the way Susanna stuck with it and was able to figure it out.

The other table groups were deeply involved and not particularly aware I was observing them. There were few difficulties and the students seemed to be making an effort to create interesting, challenging problems. All students were watching closely and checking the responses of their partners.

I returned to Susanna's group. They had abandoned their first game and started a new one. This game seemed to be running more smoothly and when it was Susanna's turn, she had less difficulty writing a related problem. Their second game went something like the following:

writer: Miguel	$1 \times 5 = 5$	solver: Kirk
writer: Kirk	$2 \times 10 = 20$	solver: Jeni
writer: Jeni	$4 \times 5 = 20$	solver: Susanna
writer: Susanna	$8 \times 10 = 80$	solver: Miguel
writer: Miguel	$12 \times 10 = 120$	solver: Kirk
writer: Kirk	$6 \times 20 = 120$	solver: Jeni
writer: Jeni	$12 \times 40 = 480$	solver: Susanna
writer: Susanna	6×40	

I noticed that another group seemed to be extremely intent on their game. Reasonably certain that Susanna and her group were fine, I went over to see what was going on. A student had written the answer of *38* for the problem 6×7. The group had been trying to convince her that the product should be 42. They had come up with three arguments before they could finally convince her to change her original answer. (See Figure 4–2.)

Gina, Jackie, and Alicia had started a game that seemed to focus on the pattern inherent in our number system when multiplying by 10, 100, and so on. They started off with 5×10 and continued until the four-teenth problem before making an error with the number of zeros that should be in the product. (See Figure 4–3.)

When I checked back with Neal, his group was intent on solving 160×240. Melissa was the one attempting to solve this problem, which Charlie had written. The other three were pointing to give her clues and supporting her by agreeing as they watched her work out the problem. After patience and support from her group, Melissa successfully completed the problem by writing the product of *38,400.* Watching Melissa work convinced me that her understanding was developing well, she was persistent, and she willingly attempted and

▲▲▲▲▲▲**Figure 4–2** *Arguments to convince a student that the product of $6 \times 7 = 42$. The error was corrected.*

▲▲▲▲▲▲**Figure 4–3** *Gina, Jackie, and Alicia focused on multiplying by 10 and multiples of 10.*

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successfully solved a problem I would have thought too difficult. (See Figure 4–4.)

I found it fascinating to watch the children solve the problems and interact with one another during this activity. I was interested to know what they thought they learned from the experience. I asked the students to respond to the following prompt: "Something I learned by playing *Silent Multiplication* is . . . " The students seemed to get a variety of things from this lesson. Jackie wrote that she learned about halving and doubling numbers and gave some examples done correctly, indicating that she understood. (See Figure 4–5.)

Susanna wrote that she understood the effects of halving and doubling on the product and gave a correct example, which really pleased me, as Susanna had had to struggle to make sense of this information.

Alicia also seemed to understand this concept and gave several nice examples as supporting evidence of her understanding. (See Figure 4–6.)

▲▲▲▲▲▲**Figure 4–5** *Jackie learned about halving and doubling.*

▲▲▲▲▲▲**Figure 4–4** *The group helped Melissa solve 160 × 240.*

▲▲▲▲▲▲**Figure 4–6** *Alicia showed her understanding.*

Melissa really enjoyed this activity. She had figured out three patterns. She wrote the patterns and gave examples of each. And, of course, she wrote about her solution to 160×240. (See Figure 4–7.)

Casey's work indicated that he thought he understood doubling. However, his examples did not support this. As his teacher, I needed to go back and talk with him about doubling and then compare it to the pattern he mentioned as an example of his understanding. (See Figure 4–8.)

Calob's work focused on his understanding of adding zeros when multiplying by 10. It was clear he recognized the pattern, but I wondered about his understanding when he explained his second example, $20 \times 20 = 400$. He indicated that he added two zeros to the sum of $2 + 2$. While adding $2 + 2$ in this example yields a correct answer because $2 + 2$ and 2×2 both equal 4, addition will not work in other cases. I needed

to work with Calob to help him recognize a couple of things: first, where the two zeros came from in 20×20, that is, 20×20 can be thought of as $2 \times 2 \times 100$ or $2 \times 2 \times 10 \times 10$, and second, why the 2s are multiplied rather than added, as he thought they should be. (See Figure 4–9.)
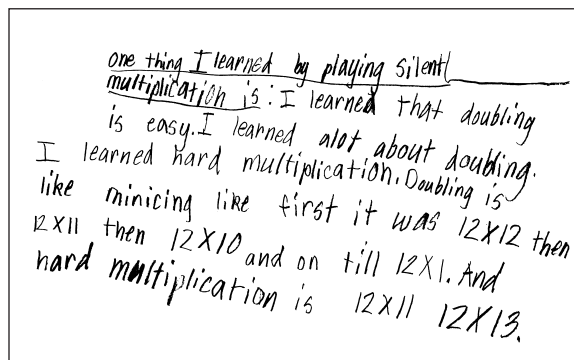
▲▲▲▲▲▲**Figure 4–8** *Casey's work indicated he needed help with the concept of doubling.*
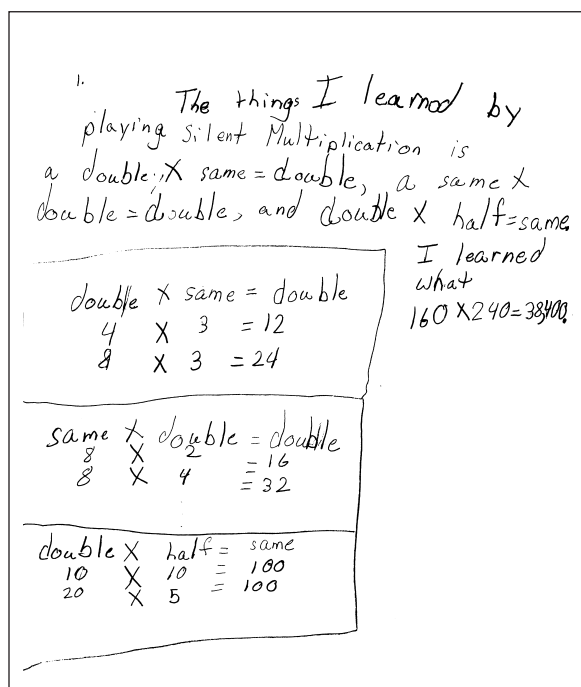
▲▲▲▲▲▲**Figure 4–7** *Melissa wrote about solving 160 × 240, a problem her group helped her solve.*
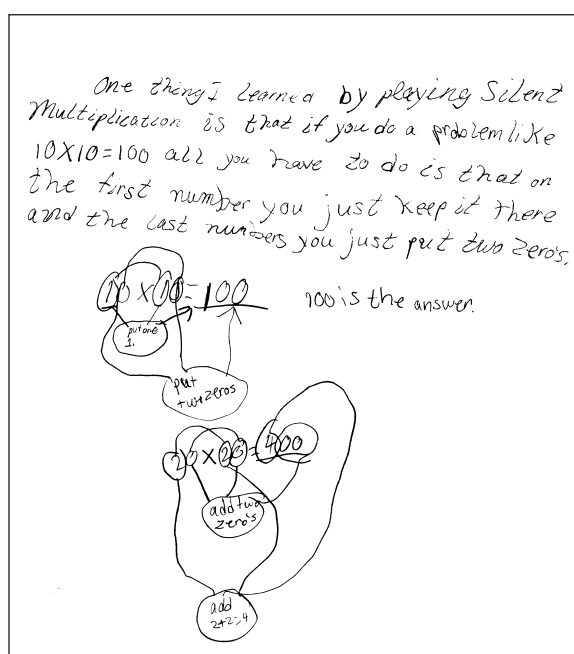
▲▲▲▲▲▲**Figure 4–9** *Calob recognized the pattern of multiplying by 10 and multiples of 10 although a misconception appears in his work.*

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Questions and Discussion

▲▲▲

▲ *Why is this activity done silently?*

The activity is done silently for several reasons. The silence seems to cause children to focus intently, more so than usual. Perhaps this is because it is different to learn silently. Perhaps it is because only one sense is being used to take in information, or maybe there is some other reason. The important thing is children are very focused. Another benefit of silence is that those children who tend to blurt out aren't as prone to that behavior. The silence seems to slow things down and give students more quality thinking and processing time.

▲ *Doesn't the silence make it difficult to correct students?*

When correcting students, I can't rely on a quick verbal response that may or may not make sense to the child. Rather, I have to think more deeply and I have found I rely more on making connections between ideas and using pictures. This is using another modality, which often benefits students.

When I have found general confusion that needed a verbal explanation, I have erased the star, indicating that talking was now OK, and we have had a discussion.

▲ *Why are the class discussions so important in this activity?*

It is during the class discussions that students explore the connections and relationships among the problems. They consider how these relationships can be used to help them solve bigger, more difficult problems. Many students will not make these connections on their own. They need the structure, questioning, and sharing to guide their thinking and develop their ability to use known information to solve a new problem.

▲ *How do you know what questions to ask?*

The questions I ask vary from situation to situation. My decisions for discussions about this activity are guided by the patterns we are exploring and by the students themselves. For example, if I want students to see the effects on the product of doubling one factor, then the problems I use and the questions I ask remain as focused as possible on this idea. I honor what students have to say, but if their comments are leading down a different path, I will often redirect the class with a question focused on the mathematics I want the students to understand. The key is to keep in mind what mathematics is to be taught while listening carefully to what students are saying and using this information to guide comments and questioning.

There are some general questions that can be used to get a discussion going. Some of these include

What did you notice?
What would happen if . . . ?
What patterns do you see?
How do you know this information is reasonable?
What did you learn or discover?
What are you still wondering about?

Once the discussion gets going, the mathematics to be explored and the students should be the guides for questions and comments.

▲ *What if my students have difficulty with doubling?*

The book *The 512 Ants on Sullivan Street,* by Carol Losi (Scholastic, 1997), provides a context through pictures and a story in which students explore the doubling pattern. In the story, the number of ants needed to take each subsequent item from the picnic basket doubles. To help students make the connection between doubling and multiplying by 2, an addition sentence can be listed representing the number of ants needed to carry away a food item. For example, four ants working in two pairs were required to carry away a chip. The addition sentence would be $2 + 2 = 4$. Next to the addition sentence, you could write a multiplication sentence representing the same situation: $2 \times 2 = 4$. This idea, along with others, appears in the back of the book.