How I Boost My Students' Number Sense

by Marilyn Burns

It's not as tough as you think! Try these strategies.

Photos: Bob Adler

Marilyn intrigues students with everyday problems that nurture number sense.

At the Chinese New Year celebration at Park School in Mill Valley, California, third graders couldn't wait for Danny's mother to teach them how to eat with chopsticks. As the anticipation grew, I said to the class, "There are 28 of us and each of us gets 2 chopsticks. Figure out in your heads how many chopsticks we need altogether." Eddie's hand shot up. "I guess 75," he volunteered. "That can't be right," Josh replied, "because the answer has to be even." Rebecca suggested, "I counted by twos and got 56." Ramón added, "I counted by twos and got a different answer." Carla agreed with Rebecca and said, "I know 25 and 25 makes 50 and we have 3 more people, so we need 6 more chopsticks. The answer is 56."

Why do some students seem to have a good grasp of numbers while others don't? What about *your* students? Is their number sense developing, and what are you doing to help it improve?

To answer these questions, you need a good handle on what number sense is and how it fits into your math program. This can be a tall order, because while the National Council of Teachers of Mathematics Standards call for teachers to give increased attention to number sense, many of us are not sure what that means. This article will help you make sense of number sense and find innovative ways to promote it in your classroom.

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What is Number Sense?

Answers to Your Questions

Q Why is number sense so hard to define?

A It's tough because number sense is a broad idea that covers a range of numerical thinking. While we can see evidence that indicates students have strong or weak number sense, we're not always clear about what they understand. Two things *are* for sure: Number sense rests on making sense of mathematical concepts and procedures, and different people reason in different ways.

Q How can I assess my students' number sense?

A Have a child explain his or her reasoning, either verbally or on paper. The information you get will only be as valuable as the questions you ask. While you can't interview all students individually, it's valuable to make time to interview a handful. Doing so is good preparation for asking questions during classroom instruction that reveal students' understanding.

Editor's Note: Marilyn models interviewing strategies in the videotape series Mathematics: Assessing Understanding *(see box, right).* *Also see "How to Assess Your Students' Number Sense," page 73.*

Q Isn't computing with paper and pencil important for students' number sense?

A You can show a child *how* to do arithmetic procedures with paper and pencil—such as adding with regrouping or multiplying fractions—but the student may not know *why* those methods make sense. Relying solely on paper-and-pencil procedures doesn't guarantee that students will develop the competence with numerical thinking that shows they have number sense.

An Instructor GIVEAWAY

Here's your chance to win a videotape of Marilyn Burns interviewing individual students to assess their number sense! The three 20-minute videos are part of a series *Mathematics: Assessing Understanding*, published by Cuisenaire Company. Part 1 deals with children ages 7 to 9; Part 2 focuses on 10-year-olds; Part 3 gives an in-depth view of a 12-year-old's understanding of fractions. For a chance to win one of these videos, send a postcard with your name, school address, and the grade you teach by May 15, 1997, to Number Sense Giveaway, Instructor, Scholastic Inc., 555 Broadway, New York, NY 10012. Ten winners will be selected at random and receive the video appropriate for the grade they teach.

Q How do basic facts come into play?

A Numerical calculations are easier, more efficient, and more convenient when we know the basic facts. But along with learning facts, students should understand *why* it's useful to know them. A focus on learning facts should always follow—not lead—the development of students' understanding.

Q Can you really teach number sense?

A There is some knowledge we acquire by being told, such as where to place silverware when setting a table or on what day Thanksgiving falls. Other ideas can't be taught by telling, but must be presented in ways that we can tumble them in our minds and construct understanding ourselves. For example, I can show young children how to write number sentences, but they must build the understanding for themselves that 6 can be taken apart and put together in many ways—3 and 3, 2 and 4, three 2s, and so on.

7 Ways to Build Number Sense

1 Link school math to real-world experiences.

As part of your daily routine, present students with problem situations that relate to their experiences, both inside and outside the classroom. This way they'll learn that numbers are useful for solving problems. Because number sense develops over time, children need regular opportunities to reason with numbers and hear others express their ideas.

Primary Activity

Pose numerical problems often that relate to classroom routines. For example, after you've figured out who is absent, ask students to calculate how many children are in the class that day. Another day, ask whether every child will have a partner if the class lines up in pairs, or have children figure out how much money a year's worth of school lunches costs for one student.

Intermediate Activity

Present problems about comparison shopping. One day I showed fifth graders two packages of raisins—a 15-ounce box that cost $1.89 and a package of 14 half-ounce snack packs that cost $1.49. I asked them which was the better deal. After talking about some possible ways to approach the problem, students wrote individual papers describing their reasoning.

2 Model different computing methods.

When children think that there is one right way to compute, they focus on learning and applying it, rather than on thinking about what method makes sense for each situation. A one-way approach in the classroom doesn't help students learn to think flexibly and develop number sense.

Intermediate Activity

Talk about division of fraction problems. With fifth graders I begin with a familiar problem—24÷3=8. "How can you prove it?" I asked. Randy said, "You count how many 3s go into 24." Others nodded. Maria said, "You can draw 24 things, like cookies, and circle 3s." I then said, "Try thinking about 3/4 divided by 1/4 as if they were whole numbers." "Oooh, I know," James said. "There are three 1/4 s in 3/4. So the answer is 3. "I got that, too," Leslie said. "I did it by drawing." She came to the board, drew a circle, divided it into fourths, and erased one wedge to leave only 3/4 of a circle. She put an X in each 1/4. "See, there are three 1/4 s in 3/4 s." We talked about making sense of other problems this way.

Doing skip counting on a 1–100 chart helps children see patterns in numbers.

3 Ask students to calculate mentally.

In our daily adult lives, while we may do numerical calculations with calculators or paper and pencil, most often we rely on mental computation to get to the movies on time, to tip in restaurants, and to put only $20 worth of food in our basket when that's all we have in our wallets.

Primary Activity

To give children experiences with computation in their heads, try what I call hands-on-the-table math. That means no paper or pencil, but only thinking and talking about a problem with others. For example, with second graders, after each of the 26 students put two "fish" (interlocking cubes) into an empty fishbowl, I asked them how they could figure out how many fish there were altogether. After listening to their ideas, I asked them to talk with a partner and record on paper their answer and how they figured it out. This assignment gave me the chance to observe them at work and then later reflect on what they had written.

4 Have class discussions about computing strategies.

Whether it's a long lesson or a short discussion, be sure students explain their reasoning. Probe the class for different ways to approach a problem. Verbalizing their reasoning helps students extend their thinking, cement ideas, and offer classmates other ways to think. Their reasoning gives you insights into how they think.

Primary Activity

After students counted 31 interlocking blue cubes in a jar and 16 orange cubes in an identical jar that was half filled, I asked, "About how many more orange cubes would I have to put into the jar to fill it." Craig and Lisa counted on their fingers, but while Lisa explained she would count from 17 to 31, Craig said he'd start with 1. Sally said she would make two long trains, one orange and one blue, and lay them next to each other. "Then I could count how many orange cubes are missing," she said. Josh suggested, "I know that 4 more makes 20, and then 10 more makes 30, and then you need 1 more." Some students were stumped. After our discussion, children wrote down the answer they thought made the most sense and explained their reasoning.

Discussing how many cubes are in the same-size jars helps children compare numbers.

Intermediate Activity

Play the game "More or Less Than 1/2." Write a fraction on the board. Ask children to tell if it's more or less than 1/2 and to explain their thinking. For 2/3, Davy said, "There are three thirds in a whole, and 2/3 is closer to a whole." Ramón volunteered, "On a measuring cup, the 2/3 line is over the 1/2 line." Leslie divided a circle into three equal-sized wedges and mused, "If you cut a cookie into thirds like this, 1/2 is 1/3 plus some more."

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Estimate, Estimate, Estimate!

As with other experiences to build number sense, estimation should be embedded in problem situations. Textbook rounding-off exercises often do not promote number sense because students don't have any context for making sense of the problem. Also, I make a distinction between estimating and guessing. When children don't have information or prior experience to draw on, they have no choice but to guess. This isn't as useful for promoting number sense as when children have some reference to guide their thinking.

Primary Activity

Show students a clear jar filled with blue interlocking cubes and ask them how many cubes they think there are. When I did this with students, I gave anyone who wanted it an opportunity to guess. Then we began removing the cubes one at a time. After we had removed 10 cubes and snapped them into a train, I asked, "Now that you have more information, how many do you think were in the jar when it was full?" I watched who used the information now available and who made wild guesses. When the jar was empty, we had three trains and one leftover cube, and counted together, "10, 20, 30, and 1 more makes 31." I wrote this on the board and replaced the cubes. I then took a jar identical to the first one and filled it halfway with orange cubes. "How many orange cubes do you think I put into the jar?" I asked. The answers varied, and I asked each child to explain his or her reasoning. Finally, we counted the orange cubes. I repeated this activity throughout the year, changing the sizes of the jars or objects.

By finding all the ways to snap 24 cubes into rectangular solids, children think numerically about the dimensions.

Intermediate Activity

Ask children what is the better deal: something that costs two for a nickel, or three for a dime. In one fourth-grade class, some students used paper and pencil, some reasoned in their heads, and others just guessed. Samantha suggested, "If you got two for a nickel, then you'd be able to buy four for a dime, and that's better than three for a dime." Brian offered, "Two for a nickel means one would cost 2 1/2 cents. Then if I bought three, that would cost a nickel and 2 1/2 cents more, and that's less than 10 cents." The students who didn't have any way to figure out the problem, or who looked confused when Brian was explaining, showed their need for more help developing number sense.

7 Ways to Build Number Sense

6 Question students about how they reason.

An important key to developing number sense is asking students to explain their thinking—at all times, not just when they make mistakes. I constantly probe, "Why do you think that? Explain why that makes sense. Tell me more about how you reasoned." Questioning students gives several important messages: you value their ideas; math is about reasoning, not just memorizing; and students should always look to make sense of mathematical problems.

Primary Activity

Ask students to create subtraction story problems that involve topics they're studying. For example, second graders at one school were studying oceans, and I said, "Write a subtraction word problem about things in the ocean. Illustrate it. Then, on another sheet of paper, explain how you solved the problem in two different ways—one using a calculator and the other by figuring out the answer without a calculator. Then you'll solve one another's problems."

7 Be sure to do plenty of measuring activities.

Problems involving measurement help build students' number sense because they can verify their estimates and calculations by actually measuring. Having a way to check their thinking in the physical world—rather than relying on the teacher or an answer book—encourages children to take risks and try new ways of thinking. "After all," I tell them, "there's no answer book in life, so you need ways to decide if your solutions to problems are reasonable."

Primary Activity

This student is learning that there's no one right way to do any problem.

Have students work in pairs to explore the lengths of different objects with interlocking cubes. Second graders in one class chose whatever objects they wanted. Some stuck to short items—pencils, rulers, books, their desks—while others tackled longer and sometimes more inaccessible objects—doors, bookcases, chalkboard tray, the classroom flagpole. For each, they were to estimate first, then measure, and then figure out how many cubes off they were. The goal was to see if their estimates got closer as they kept trying.

Intermediate Activity

If you're counting down to the 100th day of school, use a long strip of adding-machine tape posted above the chalkboard to keep track of the days by marking a one-inch interval for each school day. On the 11th day of school in one fourth-grade class, I asked the students, "How far around the room do you think the mark on our adding-machine tape will be?" After a discussion about the information they needed to figure this out, students worked in pairs. Some used rulers, yardsticks, or tape measures. Others figured first and then went to get a measuring tool. It was a lively activity.