Making Sense of the

Marilyn Burns shows you how to put the new standards in perspective—and into practice

April 2000 marks the release of new math standards, *Principles and Standards for School Mathematics*, published by the National Council of Teachers of Mathematics (NCTM). This version replaces the groundbreaking 1989 NCTM standards. What's new about the 2000 standards? What do you need to know about them? What kinds of classroom activities support them? Here, Marilyn Burns answers these and other questions. She also offers highlights—hot off the presses—from the new *Principles and Standards* and helps you use them to make smart decisions about what to teach in your classroom—and how to teach it.

TOUGH to TEACH

PHOTO: BOB ADLER

Q: I have my hands full with my daily teaching responsibilities. Why should I take time to learn about the new standards?

A: Teachers tell me all the time that it's tough to stay on top of their day-to-day duties—preparing lessons, reading student work, communicating with parents, and dealing with all the everyday problems and situations that demand attention. I can relate. From my own classroom experience, I know that there's little time for much else at the end of the day. But you can reap rewards by taking a break from daily challenges to look at the big picture as it relates to your math program. The new Principles and Standards can provide perspective and bring freshness to your teaching.

Marilyn Burns is the author of more than 20 books for teachers and children and the creator of Math Solutions, five-day summer courses offered nationwide. For information, call (800) 868-9092 or visit Math Solutions online at www.mathsolutions.com

Math Standards

Q: Why did NCTM publish new standards? What's wrong with the old ones?

A: The new NCTM Principles and Standards doesn't make dramatic changes. The important messages published in the original document in 1989 are still valid:

- ✔ Learning math is essential for students.
- ✔ All students can learn mathematics, not just those with a "gift for math."
- ✔ Learning math means more than memorizing facts and performing procedures. It means thinking, reasoning, and applying skills to problem solving.
- ✔ Students learn math by being actively involved in making sense of concepts and skills.

The new, improved Principles and Standards primarily reorganizes and clarifies the original version. In a nutshell, the current document begins by addressing six guiding principles—equity, curriculum, teaching, learning, assessment, and technology. The rest of the text focuses on ten standards. Five are content standards that define the mathematics that students must learn, and the other five are process standards that address ways for students to learn it.

Q: What's the difference between NCTM standards and my state or district standards?

A: State and district standards provide the specifics about what you are to teach. Most are aligned with the 1989 NCTM standards and, therefore, are consistent with the new version as well. The current NCTM document offers a complete and up-to-date reference that addresses the full scope of teaching mathematics from pre-K through grade 12.

Q: What are the *content standards*, and how do they outline the math students should know?

A: Content standards are the heart of what we teach:

Standard 1. Number and Operation
Standard 2. Patterns, Functions, and Algebra
Standard 3. Geometry and Spatial Sense
Standard 4. Measurement
Standard 5. Data Analysis, Statistics, and Probability

Although they're organized into five separate areas, it's important to remember that content overlaps. For example, "Measurement" helps with understanding "Number." And "Spatial Sense" plays an important role in "Patterns, Functions, and Algebra." Together, the five standards provide a way for you to be sure that your math program is addressing the full breadth of mathematics your students should know.

How to order the NEW STANDARDS

To order the National Council of Teachers of Mathematics new Principles and Standards, log on to www.nctm.org or call (800) 235-7566.

Q: What about arithmetic basics? What do the NCTM standards say about them?

A: The message about basics in Principles and Standards is loud and clear: Students need to know basic number facts and operations. Nowhere do the standards say otherwise. The goal is for all students to develop computational fluency. This means that they should be able to compute accurately and efficiently, and should also develop the number sense necessary for estimating and reasoning numerically. If anything, the basics are broadened so that computational competency goes hand in hand with understanding.

Q: I'm not comfortable with subjects like functions, statistics, and probability. What should I do?

A: You can only teach well what you understand. But it's never too late to learn, so consider finding the kind of math in-service training that can enhance your own understanding. In the November 1999 "NCTM News Bulletin," Glenda Lappan, outgoing president of NCTM, wrote an article entitled "Knowing What We Teach and Teaching What We Know." You can read it at the NCTM Web site at **www.nctm.org**

Q: What are the *process standards*, and how do they address how students should learn math?

A: They bring the content of the curriculum to life and make it accessible to children. In important ways, the two sets of standards are inseparable. The process standards are as follows:

Standard 6. Problem Solving
Standard 7. Reasoning and Proof
Standard 8. Communication
Standard 9. Connections
Standard 10. Representation

Q: How can I adapt the standards to my grade?

A: While the 1989 NCTM standards were presented in K-4, 5-8, and 9-12 grade-level spans, the new version is organized into pre-K-2, 3-5, 6-8, and 9-12. The newly organized grade-level sections provide more instructional specifics.

Also, the new Principles and Standards includes ideas for classroom teaching and examples of student work. It isn't specific enough to replace your instructional materials, but it gives a useful overview and it's a great reference guide.

Q: Do all of the standards apply to me? Some of those topics seem awfully advanced.

A: Yes. All of the content standards apply to all grades, even if you teach kindergarten. But there are differences in the emphasis that topics receive at some levels, and Principles and Standards explains this clearly.

7 Activities that meet the math standards

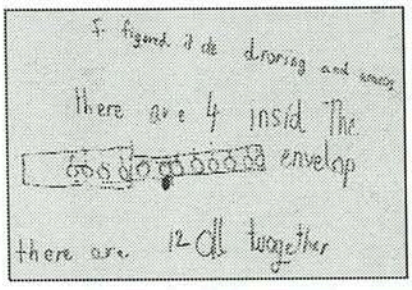
Here are some examples of activities that bring the standards into classroom instruction. All are adapted from my book 50 Problem-Solving Lessons: Grades 1-6 (Math Solutions Publications).

HOW MANY DOTS?
Supports Standard 1; and all of the process standards

GRADES K-1 **This asks young children to apply what they know about numbers to a problem-solving situation.**

Bonnie Tank presented this problem to first-graders in San Francisco, California. To prepare the lesson, she drew 12 dots on a 3" x 12" strip of tagboard and made an envelope that allowed her to slip in part of the strip while still leaving some dots showing. In class, she held up the entire strip for students to see, but didn't give them time to count the dots. Instead, she slipped the strip inside the envelope so that 4 dots were hidden and 8 were showing.

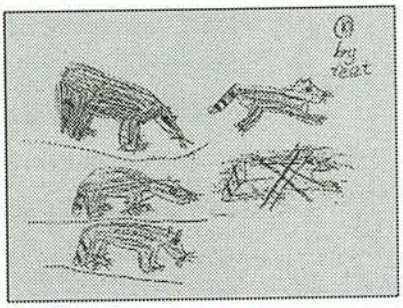
"You can't see all the dots now," she said, "because some of them are hidden. Your problem is to figure out how many dots are on the whole strip. Because you can't see the hidden dots, I'm going to give you a clue." Then she put a paper clip above the second dot showing. She pointed to the dot she marked and told them, "This is the sixth dot. Use that information to figure out how many dots there are altogether. You're to work on this problem by yourself." She then gave each child an unlined sheet of paper. "Put your answer on this paper and also explain how you figured it out. Your explanation is very important, because it tells me what you're thinking. You can also draw pictures to help you figure or explain your reasoning."



THE RACCOON PROBLEM
Supports Standard 1; and all of the process standards

GRADES K-1 **Children benefit from problem-solving experiences that help develop their number sense.**

"Today I have a problem for you to work on with partners," Bonnie Tank said to the first-graders. "First, you need to tell the problem to each other to be sure you both understand it. Then talk about what you'll write and how you'll share the work." Next, she stated the problem: Four raccoons went to the lake for a drink. Two got their front feet wet. One got its back feet wet. How many dry feet were there? (Answer: 10)



Tank talked about what the students should be writing: "What you put on your paper should help explain your thinking. It can be a drawing, numbers, or anything you think will help. You may also use blocks or counters if you'd like. When you arrive at an answer on which you both agree, put that on your paper as well."

Later, she presented these similar problems to students:

- ✔ A manufacturer needs wheels for 5 bicycles and 4 tricycles. How many wheels does he need?
- ✔ Some children went out to play in the snow. When they went back inside, they put their boots by the door to dry. There were 12 boots. How many children had gone out?
- ✔ There were 4 cows and 3 chickens in a field. How many tails and legs were there altogether?

EXPLORING HALVES
Supports Standards 1 and 3; and all of the process standards

GRADES 2-3 **This problem about fractions asks students to find different ways to divide square "cakes" into equal shares.**

David Ott taught this lesson to second-graders in Albany, California. He drew a circle on the board and asked children how he might divide it in half. After listening to responses, he drew a diameter to divide

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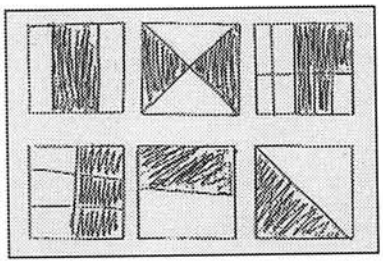
continued from page 42



the circle into two equal parts and shaded one of the halves. "You're going to work in pairs to experiment with ways to divide squares in half," he then said, holding up a sheet he had copied for each student that had been ruled into six 2 3/4" x 2 3/4" squares. "Find a different way to divide each square in half, and then shade each half of each square. Be sure that both of you can explain how you divided each square and why you're sure each part equals one-half."

After about 15 minutes, Ott asked pairs to pick one of their squares and present it to the class. As each pair came to the front of the room, Ott drew a square on the board on which the two students showed their method. The children explained why the part they shaded was one-half of the square, and classmates asked questions and challenged the results if they disagreed.

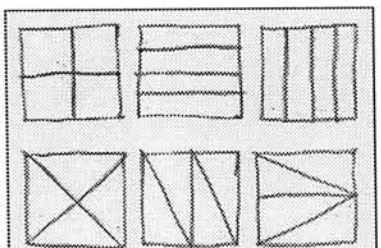
CUTTING CAKES
Supports Standards 1 and 3; and all of the process standards

GRADES 2-3

This problem gives students experience with exploring fourths and comparing areas.

Carolyn Felux asked fourth-graders in Converse, Texas, to explore ways to cut a cake into four equal pieces. She gave each child 8 1/2" x 11" sheets of paper to use as "cakes," and asked students to sketch the different ways they found to divide them. All of the children quickly found three ways to cut the cake into fourths—cutting horizontally, vertically, and diagonally. Some children explored further and found additional solutions.

This activity is similar to textbook problems that ask students to divide shapes into fourths, but Felux extended the task by asking children to respond in writing to the following directions: "Take one piece from each cake you cut. Compare the pieces to see if each one gives you the same amount of cake as the others. Explain your reasoning."



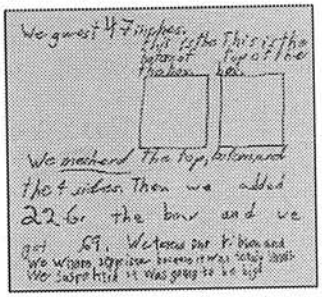
Students' papers revealed that almost half of them thought that fourths in different shapes were different sizes. These children did not understand that all pieces that are one-fourth of the same whole are the same amount. Felux might not have been aware of their misconceptions if she hadn't asked them to write about their thinking.

HOW MUCH RIBBON
Supports Standards 1 and 4; and all of the process standards

GRADES 3-4

Measurement is a wonderful vehicle for helping develop students' numerical understanding and skills.

"Today," Bonnie Tank began this lesson with a class of third-graders, "each group will get a box to wrap." She had brought to class a box for each group of four children, including shoe boxes, a shirt box, and others. "Your group's job is to figure out how many inches of ribbon you need to wrap your box, including the bow. We're going to use yarn instead of real ribbon." Tank demonstrated how to wrap yarn around the box in two directions with a bow on top. She wrote these directions on the board:



1. Talk about a plan to solve the problem.
2. Decide on the length of the yarn you need.
3. Write about how you got your answer.
4. Measure and cut yarn to test your answer.

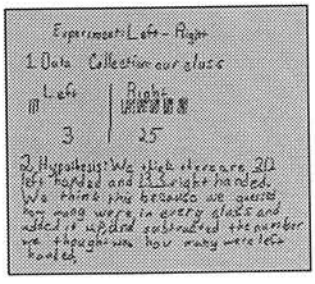
A STATISTICAL EXPERIMENT
Supports Standards 1 and 5; and all of the process standards

GRADES 3-5

This lesson gives experience with estimation; collecting, organizing, and interpreting data; making inferences; and computation.

Carolyn Felux explained to third-graders in San Antonio, Texas, that they were going to conduct an experiment. "We'll collect data about how many children in our class are right-handed and how many are left-handed," she said. "Then you'll use that information to estimate how many right-handed and left-handed students there are altogether in all the third-grade classes in the school. Please work in pairs."

There were 3 left-handed and 25 right-handed students in the class. "What information do you now need to estimate how many left-handed and right-handed third-graders there are in all the classes together?" Felux asked. As they thought of the names of the six other third-grade teachers, Felux listed them on the board. Children seemed to feel that this was all the information they needed to make their estimates, so Felux gave them further directions. "You and your partner should discuss and agree on an estimate," she said. She wrote the word hypothesis on the board. "Record your estimate and explain how you decided on it. Your explanation is your hypothesis. It should tell how you used the data to make your estimate."



continued on page 80

continued from page 44

AROUND THE EDGE

Supports Standards 1 and 2; and all of the process standards

GRADES 4–5

This problem can be solved by counting or calculating. It also encourages algebraic thinking.

"I have a problem for you to solve," I told fifth-graders in West Babylon, New York. I held up a 10 x 10 square I had cut from centimeter-squared grid paper. "How many centimeter squares do you think there are around the edge of this 10 x 10 grid?" I asked. "You could count them, but I'd like you to think about other ways to figure it out."

Working in groups of four, students found six ways to arrive at the answer of 36. I translated them into numerical expressions on the chalkboard.

$(8 \times 4) + 4 = 36$

$10 + 10 + 8 + 8 = 36$

$(10 \times 4) - 4 = 36$

$10 + 9 + 9 + 8 = 36$

$9 \times 4 = 36$

$100 - 64 = 36$

We then discussed how the methods they used would apply to a 5 x 5 grid, a 4 x 4 grid, and a 3 x 3 grid. "I'm going to write a formula to describe one method," I told the class. "I'll use S to stand for the number of squares on one side and E to stand for the number of squares around the edge. See if you can tell which method it is." I wrote on the board:

$S + S + (S - 2) + (S - 2) = E$

It seemed obvious to the students that the formula I wrote described the second method. I then gave the class another problem. "Your job is to write a formula for each of the other methods," I said. ■